UC Irvine

UC Irvine Previously Published Works

Title

Real part of the K+p amplitude based on duality

Permalink

https://escholarship.org/uc/item/93f3m9dn

Journal

Physical Review D, 8(9)

ISSN

0556-2821

Authors

Bander, M
Stone, TM

Publication Date

1973

DOI

10.1103/PhysRevD.8.3203

Copyright Information

This work is made available under the terms of a Creative Commons Attribution License, availalbe at https://creativecommons.org/licenses/by/4.0/

Peer reviewed

eScholarship.org Powered by the California Digital Library
University of California

Real Part of the K^+p Amplitude Based on Duality*

Myron Bander and Theodore M. Stone
Department of Physics, University of California, Irvine, California 92664
(Received 9 April 1973)

The concept of duality is applied to the real part of K^+p scattering. We study this part of the amplitude via fixed-t dispersion relations. We are interested in seeing what effect replacing the left-hand discontinuities (K^-p scattering) by a low-energy extrapolation of Regge-pole amplitudes will have on the real part or the right-hand side (K^+p scattering).

In this note we continue the test of how well duality, specifically two-component duality,[1] describes low-energy K^+p elastic scattering. In a previous communication[2] we showed that the imaginary part of the A' amplitude in K^+p scattering was well represented by the contribution of a Pomeranchuk-trajectory exchange in the t channel down to $\nu \sim 1.4$ GeV and for nonforward directions, $0 \leqslant |t| \leqslant 0.6$ GeV^2. Presently we wish to investigate how duality bears on the real part of this amplitude.

We write a fixed-t dispersion relation for the A' amplitude (possibly subtracted):

$$A'(\nu,t) = \text{poles} + \frac{1}{\pi}\int_{\nu_0}^{\infty} \frac{\mathrm{Im}A'(\nu',t)}{\nu'-\nu}\,d\nu' + \frac{1}{\pi}\int_{-\infty}^{-\nu_0} \frac{\mathrm{Im}A'(\nu',t)}{\nu'-\nu}\,d\nu' \,, \tag{1}$$

with

$$\begin{aligned} \nu &= E_{\text{lab}} + t/4M, \\ \nu_0 &= M_K \,, \\ \nu_0 &= [(M_\Lambda + M_\pi)^2 - M^2 - M_K{}^2 + t]/2M \,. \end{aligned} \tag{2}$$

We note that K^+p is an exotic channel; two-component duality tells us that the imaginary part of this amplitude on the right ($\nu > M_K$) can be averaged by a Pomeranchuk contribution. In Ref. 2 we found this hypothesis verified for a trajectory with a slope $\alpha_P' \sim 0.5$ GeV^{-2}. The left-hand integral of the dispersion relation is related to K^-p scattering and its imaginary part is not as simple. Besides a background whose imaginary part is averaged by a Pomeranchuk trajectory, there are resonances whose contribution is averaged by the subsidiary Regge trajectories. This smoothing out of the local fluctuations in the imaginary part of the amplitude would not be valid were we interested in evaluating the real part of the K^-p amplitude. It is hoped that this averaging is sufficiently good that the influence of the imaginary part of the K^-p scattering amplitude on the real part of the K^+p scattering amplitude may be sufficiently well described by such a procedure.

The Regge-pole approximation for the A' amplitude for K^+p scattering was written as

$$\overline{A}'(\nu,t) = \frac{\beta_P(t)(\nu_0{}^2-\nu^2)^{\alpha_P(t)/2}}{\sin\pi\alpha_P(t)} + \frac{\beta_R(t)(\nu_1+\nu)^{\alpha_R(t)}}{\sin\pi\alpha_R(t)} \tag{3}$$

with $\alpha_P(t) = 1 + 0.5t$ and $\beta_P(t)$ determined from Ref. 2. The second term represents the combined contribution of the vector-tensor complex (ρ, ω, f^0, A_2); its contribution is purely real for K^+p scattering and develops an imaginary part in the K^-p region ($\nu < -\nu_1$). $\alpha_R(t)$ was parametrized by $\alpha_R(t) = \alpha_0 + t$; α_0, ν_1, and $\beta(t)$ are parameters whose values and significance are discussed below.

The hypothesis we wish to test is whether the following approximation to Eq. (1) is valid:

$$\mathrm{Re}A'(\nu,t) = \mathrm{Re}\overline{A}'(\nu,t) + \frac{P}{\pi}\int_{\nu_0}^{\nu_{\max}} \frac{\mathrm{Im}A'(\nu',t) - \mathrm{Im}\overline{A}'(\nu',t)}{\nu'-\nu}\,d\nu' \,. \tag{4}$$

The integral on the right-hand side above takes into account the fact that the Pomeranchuk trajectory does not describe the imaginary part of the K^+p amplitude exactly. As $\beta_R(t)$ is *a priori* unknown, we turn Eq. (4) around to obtain

$$\left\{\mathrm{Re}A'(\nu,t) - \frac{P}{\pi}\int_{\nu_0}^{\nu_{\max}} \frac{\mathrm{Im}[(A'(\nu',t) - \overline{A}'(\nu',t)]}{\nu'-\nu}\,d\nu'\right\} \frac{\sin\pi\alpha_R(t)}{(\nu_1+\nu)^{\alpha_R(t)}} = \beta_R(t) \,. \tag{5}$$

The crucial test is to what degree the above expression is independent of ν. Secondly, the residue function, $\beta_R(t)$, may be compared to the one obtained from high-energy Regge phenomenology.

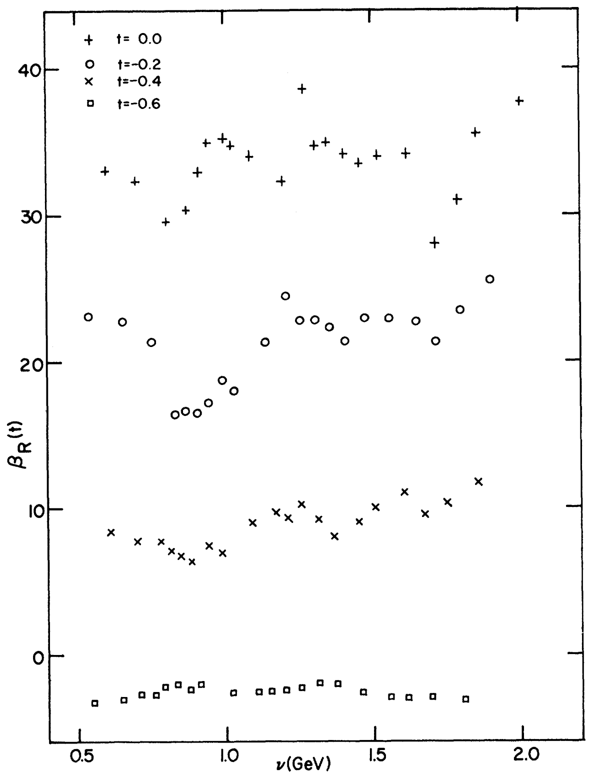

FIG. 1. Residue functions for the effective ρ-ω-f^0-A_2 trajectory obtained in this analysis.

ν_{max} in the above is dictated by the limits of phase-shift analysis. In this study we used the fit of Albrow *et al.*(3) which extends to $E_{lab}\sim 2.4$ GeV. To avoid artificial fluctuations in the principal-value integral for $\nu\sim\nu_{max}$ the analysis was restricted to $E_{lab}<2.0$ GeV. The choice of ν_1 in Eq. (5) is to some extent arbitrary. We expect the imaginary part of $\bar{A}'$ for ν negative to average the imaginary part of the true amplitude. It thus seems reasonable that the region $\nu<\nu_1$ should include all the discontinuities of the K^-p amplitude. The lowest-mass discontinuity being the Λ pole, $\nu_\Lambda\sim 0$, it is likely that $\nu_1\sim 0$. Though we varied ν_1 between 0 and m_K, the best results obtained and the ones we shall discuss have been for $\nu_1=0$.

With ν_1 and ν_{max} fixed it remains to be seen if there is an α_0 for the vector-tensor trajectories such that Eq. (5) is satisfied. In Fig. 1 we see that for $\alpha_0=0.55$, in spite of the scatter in the data, the result are relatively constant in ν for E_{lab} from just above threshold to 2.0 GeV. We did not pursue the analysis for $|t|>0.6$ GeV^2 as the phase-shift analysis becomes unreliable. The scatter around $\nu\sim 1$ GeV may be attributed to the "Cool bump,"(4) where the Pomeranchuk contribution and the true imaginary part disagree most.

We may now compare the residue function, $\beta_R(t)$, obtained in this analysis with that which one finds in fitting high-energy scattering data.(5) As our analysis forces β_R to go through zero at $t\sim -0.6$ GeV^2 consistent with the high-energy analysis, we only have to study the value of $\beta_R(0)$. From Fig. 1 we note that $\beta_R(0)\approx 34$. This value is about 15% lower than $\beta_R(0)$ obtained from, for example, the energy variation of $\sigma_{total}(K^-p)$.[5] We have no ready explanation for this discrepancy, if indeed, within the context of the approximations made it is a discrepancy.

The present work has a bearing on other works(6) where duality and dispersion relations were used to study the real parts of scattering amplitudes. However, in these articles the interest was in the real part at high energies rather than at the lower ones studied presently. Of more interest is the comparison with the interference model of Coulter, Ma, and Shaw.(7) In this model the imaginary part of an amplitude and its effect on the real part are treated "exactly," while the real part due to the crossed channels is replaced by an extrapolation to lower energies of the high-energy Regge parameters. This interference model was successful in describing data in the region where Regge analysis and resonances overlapped. In the present work we have shown that it gives an adequate description of the low-energy K^+p scattering.

We wish to thank Dr. T. C. Sens for sending us a thorough list of K^+p phase shifts.

---

*Work supported in part by the National Science Foundation.

[1]P. Freund, Phys. Rev. Lett. **20**, 235 (1968); H. Harari, *ibid.* **20**, 1395 (1968); H. Harari and Y. Zarmi, Phys. Rev. **187**, 2230 (1969).
[2]M. Bander and T. Stone, Phys. Rev. D **4**, 248 (1971).
[3]M. G. Albrow *et al.*, Nucl. Phys. **B30**, 273 (1971). The results are presented for solution γ.
[4]R. Cool *et al.*, Phys. Rev. Lett. **17**, 102 (1966).
[5]E. Berger and G. Fox, Phys. Rev. **188**, 2120 (1969).
[6]M.-S. Chen and F. Paige, Phys. Rev. D 5, 2760 (1972); M. Coirier, J. Guillaume, Y. Leroyer, and Ph. Salin, Nucl. Phys. **B44**, 157 (1972).
[7]P. Coulter, E. Ma, and G. Shaw, Phys. Rev. Lett. **23**, 106 (1969); E. Ma and G. Shaw, Phys. Rev. D 3, 1264 (1971).